Journal of Statistics: Advances in Theory and Applications
Volume 11, Number 1, 2014, Pages 45-70

WHY IS THE NULL HYPOTHESIS REJECTED FOR "ALMOST EVERY" INFINITE SAMPLE BY SOME HYPOTHESIS TESTING OF MAXIMAL RELIABILITY?

G. PANTSULAIA and M. KINTSURASHVILI

Vekua Institute of Applied Mathematics
Tbilisi State University
Tbilisi DC 0143
Georgia
e-mail: gogipantsulaia@yahoo.com

Department of Mathematics
Georgian Technical University
Tbilisi DC 0175
Georgia

Abstract

The notion of a Haar null set introduced by Christensen in 1973 and reintroduced in 1992 in the context of dynamical systems by Hunt, Sauer and Yorke, has been used, in the last two decades, in studying exceptional sets in diverse areas, including analysis, dynamic systems, group theory, and descriptive set theory. In the present paper, the notion of "prevalence" is used in studying the properties of some infinite sample statistics and in explaining why the null hypothesis is sometimes rejected for "almost every" infinite sample by some hypothesis testing of maximal reliability. To confirm that the conjectures of Jum Nunnally [17] and Jacob Cohen [5] fail for infinite samples, examples of

2010 Mathematics Subject Classification: Primary 91B02, 91B06, 91B70; Secondary 28C10, 43A46.

Keywords and phrases: a linear one-dimensional stochastic model, null hypothesis, an objective infinite sample well-founded estimate, shy set, prevalence.

Received December 24, 2013; Revised February 11, 2014

the so called objective and strong objective infinite sample well-founded estimate of a useful signal in the linear one-dimensional stochastic model are constructed.

1. Introduction

Criticism of statistical Hypothesis Testing can be found in [15], [18], [4], [8], [12], [14] citing 300-400 primary references. Much of philosophical criticism of Hypothesis Testing is based on a general opinion that the theory of mathematical statistics and the results of testing turn out inconsistent in many situations. Different reasonable statistical methods lead to inconsistent decisions in many expensive experiments, which causes the feeling of alarm in mathematicians and statisticians. It would be practically impossible to explain all the paradoxes which underlie the existence of a big gap between theory and practice. The present paper skips the detailed consideration of the issues which give grounds for such criticism. The attention is focused on a certain confusion, which is noticed in the works of Jum Nunnally [17] and Jacob Cohen [5]. In [5], Jacob Cohen says: "... *don't look for a magic alternative to NHST* [*null hypothesis significance testing*] *... It does not exist*." In [17], Jum Nunnally conjectured: "*If the decisions are based on convention they are termed arbitrary or mindless*, *while those not so based may be termed subjective. To minimize type II errors*, *large samples are recommended. In psychology*, *practically all null hypotheses are claimed to be false for sufficiently large samples so ... it is usually nonsensical to perform an experiment with the sole aim of rejecting the null hypothesis*".

Therefore, there naturally arises a question whether the concept of a theory of statistical decisions can be introduced for infinite samples and whether the conjectures made by Jacob Cohen and Jum Nunnally for sufficiently large samples are also valid for infinite samples. To confirm that these conjectures no always hold in the case of infinite samples, for a linear one-dimensional stochastic system, we consider a certain Hypothesis Testing for infinite samples, such that the sum of errors of types I and II is equal to zero (we describe such tests as tests of maximal reliability). Furthermore, we explain why the null hypothesis is claimed to be false for "almost every" [9] infinite sample by using well known infinite sample well-founded estimates. Using the result of the paper [19], we construct an example of such an infinite sample well-founded estimate for which there does not exist a null hypothesis, which is accepted or rejected for "almost every" infinite sample. This example motivates to introduce a new class of infinite sample objective well-founded estimates of a useful signal. Notice that for a hypothesis testing equipped with an objective estimate the above mentioned conjectures of Jum Nunnally [17] and Jacob Cohen [5] fail.

To begin with, notice that in many cases even the information that some phenomenon occurs with probability 1 can be quite poor and we may regard this as an essential reason for all inconsistent statistical decisions. Indeed, let X be an infinite-dimensional topological vector space. Let P be any sentence formulated for elements in X and let μ be any probability Borel measure on X. Let us discuss what information the following sentence implies:

"μ-*almost every element of X satisfies the property P.*"

If X is separable, then an arbitrary non-zero σ-finite Borel measure defined on X is concentrated on the union of countable compact subsets $(F_k)_{k\in\mathbb{N}}$ in B (cf. [11]) and for arbitrary $k \in \mathbb{N}$, there exists a vector $v_k \in X$, which spans the line L_k, such that every translation of L_k meets F_k at one point at most. Thus, the support of μ can be assumed to be the union of a countable family of "surfaces". Hence, the information described by the above sentence can, in general, be very poor. That is why, it is not advisable to study the behaviour of various general systems defined in infinite-dimensional separable topological vector spaces in terms of some partial σ-finite Borel measure (for example, a Gaussian measure concentrated on a poor set) and we need to extend the measure theoretic terms to the terms "measure zero" and "almost every". This phenomenon was originally noticed by Christensen [2] and, more recently, by Hunt et al. [9]; Mycielski [16]; Dougherty [6] and other mathematicians. Like the concept of “Lebesgue almost every” on finite-dimensional spaces, their notion of “prevalence” is translation invariant. Instead of using a specific measure on the entire space, they define prevalence in terms of the class of all probability measures with compact support. Prevalence is a more appropriate condition than the topological concepts of “open and dense” or “generic” when one desires a probabilistic result on the likelihood of a given property on a function space.

The aims of the present paper are:

● to apply the “almost every” approach to the study the properties of infinite sample statistics;

● to introduce concepts of “subjective” and “objective” infinite sample well-founded estimates of a useful signal in a linear one-dimensional stochastic model;

● to show that each infinite sample well-founded estimate of a useful signal in a linear one-dimensional stochastic model is “subjective” or “objective”;

● to show that the conjectures of Jum Nunnally [17] and Jacob Cohen [5] hold for “subjective” infinite sample well-founded estimates;

● to show that the conjectures of Jum Nunnally [17] and Jacob Cohen [5] fail for “objective” infinite sample well-founded estimates.

The rest of the paper is the following:

Section 2 presents some auxiliary notions and facts from functional analysis and measure theory.

Section 3 considers well-founded estimates of a useful signal in the linear one-dimensional stochastic model.

Section 4 contains several examples of hypothesis testing of maximal reliability for a linear one-dimensional stochastic model and an explanation why the null hypothesis is rejected for “almost every” infinite sample by some hypothesis testing of maximal reliability.

Section 5 presents the partition of the class of all infinite sample well-founded estimates into pairwise-disjoint two classes one of them consists only subjective and another of them consists only objective estimates. Here is given an explanation why the application of objective statistics is more appropriate than an application of subjective ones in the statistical decision theory.

2. Auxiliary Notions and Facts from Functional Analysis and Measure Theory

Let $\mathbb{V}$ be a complete metric linear space, by which we mean a vector space (real or complex) with a complete metric for which the operations of addition and scalar multiplication are continuous. When we speak of a measure on $\mathbb{V}$, we will always mean a nonnegative measure that is defined on the Borel sets of $\mathbb{V}$ and is not identically zero. We write $S + v$ for the translation of a set $S \subseteq \mathbb{V}$ by a vector $v \in \mathbb{V}$.

Definition 2.1 ([9], Definition 1, p. 221)**.** A measure μ is said to be transverse to a Borel set $S \subset \mathbb{V}$, if the following two conditions hold:

(i) There exists a compact set $U \subset \mathbb{V}$ for which $0 < \mu(U) < 1$.

(ii) $\mu(S + v) = 0$ for every $v \in \mathbb{V}$.

Definition 2.2 ([9], Definition 2, p. 222; [1], p. 1579)**.** A Borel set $S \subset \mathbb{V}$ is called shy, if there exists a measure transverse to S. More generally, a subset of $\mathbb{V}$ is called shy if it is contained in a shy Borel set. The complement of a shy set is called a prevalent set. We say that a set is Haar ambivalent if it is neither shy nor prevalent.

Definition 2.3 ([9], p. 226)**.** We say “almost every” element of $\mathbb{V}$ satisfies some given property, if the subset of $\mathbb{V}$ on which this property holds is prevalent.

Lemma 2.4 ([9], Fact 3″, p. 223)**.** *The union of a countable collection of shy sets is shy*.

Lemma 2.5 ([9], Fact 8, p. 224)**.** *If* $\mathbb{V}$ *is infinite dimensional*, *all compact subsets of* $\mathbb{V}$ *are shy*.

Definition 2.6 ([9], Definition 6, p. 225)**.** We call a finite-dimensional subspace $P \subset \mathbb{V}$ a probe for a set $T \subset \mathbb{V}$, if the Lebesgue measure supported on P is transverse to a Borel set which contains the complement of T.

Remark 2.7. Notice that a sufficient (but not a necessary) condition for T to be prevalent is for it to have a probe.

One can consult [9] in order to see whether by constructing appropriate probes the validity of the following assertions can be obtained.

Example 2.8 ([9], Proposition 1, p. 226)**.** "Almost every" function $f : [0; 1] \to \mathbb{R}$ in L_1 satisfies $\int_0^1 f(x)dx \neq 0$.

Example 2.9 ([9], Proposition 2, p. 226)**.** For $1 < p \leq \infty$, "almost every" sequence $(a_i)_{i \in \mathbb{N}}$ in ℓ^p has the property that $\sum_{i=1}^{\infty} a_i$ diverges.

Example 2.10 ([9], Proposition 4, p. 226)**.** "Almost every" continuous function $f : [0, 1] \to \mathbb{R}$ is nowhere differentiable.

Lemma 2.11 ([11], Lemma 2, p. 58)**.** *Let* μ *be a Borel probability measure defined in a complete separable metric space* $\mathbb{V}$. *Then*, *there exists a countable family of compact sets* $(F_k)_{k \in \mathbb{N}}$ *in* $\mathbb{V}$ *such that*

$$\mu(\mathbb{V} \setminus \cup_{k \in \mathbb{N}} F_k) = 0.$$

Let $\mathbb{R}^{\mathbb{N}}$ and $\mathbb{R}^{(\mathbb{N})}$ be a vector subspace of the infinite-dimensional topological vector space of all real-valued sequences equipped with the product topology and a vector space of all eventually zero sequences, respectively.

Lemma 2.12 ([19], Theorem 1, p.79)**.** *There exists a family* $(C_t)_{t\in\mathbb{R}}$ *of Borel subsets in* $\mathbb{R}^{\mathbb{N}}$ *such that the following four conditions are satisfied*:

(a) $C_t + \mathbb{R}^{(\mathbb{N})} = C_t$ *for* $t \in \mathbb{R}$.

(b) $(C_t)_{t\in\mathbb{R}}$ *is a partition of* $\mathbb{R}^{\mathbb{N}}$.

(c) C_t *is ambivalent for* $t \in \mathbb{R}$.

(d) E*very translate of* C_s *intersects* C_t *in a shy set for different* $s, t \in \mathbb{R}$.

3. Some Well-founded Estimates in the Linear One-dimensional Stochastic Model

Suppose that Θ is a vector subspace of the infinite-dimensional topological vector space $\mathbb{R}^{\mathbb{N}}$.

In the information transmission theory, we consider the linear one-dimensional stochastic system

$$(\xi_k)_{k\in\mathbb{N}} = (\theta_k)_{k\in\mathbb{N}} + (\Delta_k)_{k\in\mathbb{N}}, \tag{3.1}$$

where $(\theta_k)_{k\in\mathbb{N}} \in \Theta$ is a sequence of useful signals, $(\Delta_k)_{k\in\mathbb{N}}$ is sequence of independent identically distributed random variables (the so-called generalized "white noise") defined on some probability space $(\Omega, \mathcal{F}, P)$ and $(\xi_k)_{k\in\mathbb{N}}$ is a sequence of transformed signals. Let μ be a Borel probability measure on $\mathbb{R}$ defined by a random variable Δ_1. Then the $\mathbb{N}$-power of the measure μ denoted by $\mu^{\mathbb{N}}$ coincides with the Borel probability measure on $\mathbb{R}^{\mathbb{N}}$ defined by the generalized "white noise", i.e.,

$$(\forall X)(X \in \mathcal{B}(\mathbb{R}^{\mathbb{N}}) \to \mu^{\mathbb{N}}(X) = P(\{\omega : \omega \in \Omega \,\&\, (\Delta_k(\omega))_{k\in\mathbb{N}} \in X\})), \tag{3.2}$$

where $\mathcal{B}(\mathbb{R}^{\mathbb{N}})$ is the Borel σ-algebra of subsets of $\mathbb{R}^{\mathbb{N}}$.

In the information transmission theory, a general decision is that the Borel probability measure λ, defined by the sequence of transformed signals $(\xi_k)_{k\in\mathbb{N}}$ coincides with $(\mu^{\mathbb{N}})_{\theta_0}$ for some $\theta_0 \in \Theta$ provided that

$$(\exists\theta_0)(\theta_0 \in \Theta \to (\forall X)(X \in \mathcal{B}(\mathbb{R}^{\mathbb{N}}) \to \lambda(X) = (\mu^{\mathbb{N}})_{\theta_0}(X))), \tag{3.3}$$

where $(\mu^{\mathbb{N}})_{\theta_0}(X) = \mu^{\mathbb{N}}(X - \theta_0)$ for $X \in \mathcal{B}(\mathbb{R}^{\mathbb{N}})$.

Here, we consider a particular case of the above model (3.1) when a vector space of useful signals Θ has the form

$$\Theta = \{(\theta, \theta, \cdots) : \theta \in \mathbb{R}\}. \tag{3.4}$$

For $\theta \in \mathbb{R}$, a measure $\mu_\theta^{\mathbb{N}}$ defined by

$$\mu_\theta^{\mathbb{N}} = \mu_\theta \times \mu_\theta \times \cdots,$$

where μ_θ is a θ-shift of μ (i.e., $\mu_\theta(X) = \mu(X - \theta)$ for $X \in \mathcal{B}(\mathbb{R})$), is called the $\mathbb{N}$-power of the θ-shift of μ on $\mathbb{R}$. It is obvious that $\mu_\theta^{\mathbb{N}} = (\mu^{\mathbb{N}})_{(\theta, \theta, \cdots)}$.

Using the concepts of the theory of statistical decisions, a triplet $(\mathbb{R}^{\mathbb{N}}, \mathcal{B}(\mathbb{R}^{\mathbb{N}}), \mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$ is called a statistical structure describing the linear one-dimensional stochastic system (3.1).

Definition 3.1. A family of Borel measurable functions $T_n : \mathbb{R}^n \to \mathbb{R}$ $(n \in \mathbb{N})$ is called a well-founded estimate of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$, if the condition

$$\mu_\theta^{\mathbb{N}}(\{(x_k)_{k\in\mathbb{N}} : (x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \,\&\, \lim_{n\to\infty} T_n(x_1, \ldots, x_n) = \theta\}) = 1, \tag{3.5}$$

holds for each $\theta \in \mathbb{R}$.

Lemma 3.2. *Let F be a strictly increasing continuous distribution function on $\mathbb{R}$ and μ be a Borel probability measure on $\mathbb{R}$ defined by F. For $\theta \in \mathbb{R}$, we set $F_\theta(x) = F(x - \theta)(x \in \mathbb{R})$ and denote by μ_θ the Borel probability measure on $\mathbb{R}$ defined by $\mathbb{F}_\theta$ (obviously, this is an equivalent definition of the θ-shift of μ). Then a function $T_n : \mathbb{R}^n \to \mathbb{R}$ defined by*

$$T_n(x_1, \cdots, x_n) = -F^{-1}(n^{-1}\#(\{x_1, \cdots, x_n\} \cap (-\infty; 0])), \tag{3.6}$$

for $(x_1, \ldots, x_n) \in \mathbb{R}^n (n \in \mathbb{N})$, is a well-founded estimate of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta \in \mathbb{R}}$.

Definition 3.3. Following [10], the family $(\mu_\theta^{\mathbb{N}})_{\theta \in \mathbb{R}}$ is called strongly separated in the usual sense, if there exists a family $(Z_\theta)_{\theta \in \mathbb{R}}$ of Borel subsets of $\mathbb{R}^{\mathbb{N}}$ such that

(i) $\mu_\theta^{\mathbb{N}}(Z_\theta) = 1$ for $\theta \in \mathbb{R}$;

(ii) $Z_{\theta_1} \cap Z_{\theta_2} = \emptyset$ for all different parameters θ_1 and θ_2 from $\mathbb{R}$;

(iii) $\bigcup_{\theta \in \mathbb{R}} Z_\theta = \mathbb{R}^{\mathbb{N}}$.

Definition 3.4. Following [10], an expanded Borel quasi-measurable [1] function $T : \mathbb{R}^n \to \overline{\mathbb{R}}$ is called an infinite sample well-founded estimate of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta \in \mathbb{R}}$, if the following condition:

$$(\forall\theta)(\theta \in \mathbb{R} \to \mu_\theta^{\mathbb{N}}(\{(x_k)_{k \in \mathbb{N}} : (x_k)_{k \in \mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \ \&\ T((x_k)_{k \in \mathbb{N}} = \theta\}) = 1) \tag{3.7}$$

is fulfilled, where $\overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$.

[1]A function $T : \mathbb{R}^n \to \overline{\mathbb{R}}$ is called an expanded Borel quasi-measurable if $f^{-1}(x) \in \mathcal{B}(\mathbb{R}^{\mathbb{N}})$ for each $x \in \overline{\mathbb{R}}$.

Remark 3.5. The existence of an infinite sample well-founded estimate of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$ implies that the family $(\mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$ is strongly separated in a usual sense. Indeed, if we set $Z_\theta = \{(x_k)_{k\in\mathbb{N}} : (x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \ \& \ T((x_k)_{k\in\mathbb{N}} = \theta\}$ for $\theta \in \mathbb{R}$, then all the conditions of Definition 3.3 will be satisfied.

By the strong law of large numbers, one can easily obtain the validity of the following assertion:

Lemma 3.6. *Let F be a strictly increasing continuous distribution function on $\mathbb{R}$ and μ be the Borel probability measure on $\mathbb{R}$ defined by F. Suppose that the first order absolute moment of μ is finite and the first order moment of μ is equal to zero. For $\theta \in \mathbb{R}$, we set $F_\theta(x) = F(x-\theta)(x \in \mathbb{R})$ and denote by μ_θ the Borel probability measure on $\mathbb{R}$ defined by F_θ. Then the estimates $\overline{\lim}\widetilde{T_n} := \inf_n \sup_{m\geq n} \widetilde{T_m}$ and $\underline{\lim}\widetilde{T_n} := \sup_n \inf_{m\geq n} \widetilde{T_m}$ are infinite sample well-founded estimates of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$, where $\widetilde{T_n} : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is defined by*

$$(\forall (x_k)_{k\in\mathbb{N}})((x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \to \widetilde{T_n}((x_k)_{k\in\mathbb{N}}) = n^{-1}\sum_{k=1}^{n} x_k). \tag{3.8}$$

Lemma 3.7 ([20], Theorem 4.2, p. 483)**.** *Let F be a strictly increasing continuous distribution function on $\mathbb{R}$ and μ be the Borel probability measure on $\mathbb{R}$ defined by F. For $\theta \in \mathbb{R}$, we set $F_\theta(x) = F(x-\theta)(x \in \mathbb{R})$ and denote by μ_θ the Borel probability measure on $\mathbb{R}$ defined by F_θ. Then the estimates $\overline{\lim}\widetilde{T_n} := \inf_n \sup_{m\geq n} \widetilde{T_m}$ and $\underline{\lim}\widetilde{T_n} := \sup_n \inf_{m\geq n} \widetilde{T_m}$ are infinite sample well-founded estimates of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$, where $\widetilde{T_n} : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is defined by*

$$(\forall (x_k)_{k\in\mathbb{N}})((x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \to \widetilde{T_n}((x_k)_{k\in\mathbb{N}})$$
$$= -F^{-1}(n^{-1}\#(\{x_1, \cdots, x_n\} \cap (-\infty; 0]))). \tag{3.9}$$

Remark 3.8. By Remark 3.5 and Lemmas 3.6-3.7, we deduce that the families of powers of shift measures described in the corresponding lemmas are strongly separated in the usual sense.

4. An Application of "Almost Every" Approach to the Study the Properties of the Hypothesis Testing of Maximal Reliability

Let us recall some notions of the theory of statistical decisions.

Let $(\mathbb{R}^{\mathbb{N}}, \mathcal{B}(\mathbb{R}^{\mathbb{N}}), \mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$ be a statistical structure described the linear one-dimensional stochastic system (3.1).

Definition 4.1. Let a null hypothesis H_0 be defined by $H_0 : \theta = \theta_0$, where $\theta \in \mathbb{R}$. A triplet (T, U_0, U_1), where

(i) $T : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is a statistic (equivalently, an expanded Borel quasi-measurable function),

(ii) $U_0 \cup U_1 = \mathbb{R}^{\mathbb{N}}$, $U_0 \cap U_1 = \emptyset$ and $U_0 \in \mathcal{B}(\mathbb{R}^{\mathbb{N}})$,

is called a statistical test (criterion) for acceptance of null hypothesis H_0 (or equivalently, HT (hypothesis testing)).

For an infinite sample $x \in \mathbb{R}^{\mathbb{N}}$, we accept the null hypothesis H_0 if $T(x) \in U_0$ and reject it, otherwise.

T is called test statistic of the criterion (T, U_0, U_1).

U_0 is called the region of acceptance for the null hypothesis H_0.

U_1 is called the region of rejection (equivalently, the critical region) for the null hypothesis H_0.

Definition 4.2. A decision obtained by the criterion (T, U_0, U_1) is called an error of type I, if the null hypothesis H_0 has been rejected whenever the null hypothesis H_0 was true.

Definition 4.3. A decision obtained by the criterion (T, U_0, U_1) is called an error of type II, if the null hypothesis H_0 has been accepted whenever null hypothesis H_0 was false.

Definition 4.4. The value

$$\mu_\theta^{\mathbb{N}}(\{x : T(x) \in U_1 | H_0\}) = \alpha, \tag{4.1}$$

is called the size (equivalently, the significance level) of the test T.

Definition 4.5. The value

$$\mu_\theta^{\mathbb{N}}(\{x : T(x) \in U_0 | H_1\}) = \beta, \tag{4.2}$$

is called the power of the test T.

In many cases, it is not possible to reduce values α and β simultaneously. For this reason, we fix the probability α of an error of type I and consider the critical regions U_1 for which the following condition:

$$\mu_\theta^{\mathbb{N}}(\{x : T(x) \in U_1 | H_0\}) \leq \alpha \tag{4.3}$$

holds. Further, among such critical regions, we choose a region U_1^* for which the error of type II is maximal.

We will see that for model (3.1), there exists a rich class of statistical tests for which $\alpha + \beta = 0$. Further, we will try to explain in terms "almost every" introduced in Definition 2.3 why the application of some of them leads us to confusion provided that there always is a null hypothesis which is rejected or accepted for "almost every" infinite sample by the corresponding test statistic.

Example 4.6. Let us consider the linear one-dimensional stochastic system (3.1) for which F is a linear standard Gaussian (or Cauchy) distribution function on $\mathbb{R}$.

For $\theta \in \mathbb{R}$, we put

$$D_\theta = \{ (x_k)_{k\in\mathbb{N}} : (x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \quad \& \quad \overline{\lim}\widetilde{T_n}((x_k)_{k\in\mathbb{N}}) = \theta \}, \quad (4.4)$$

where the estimate $\overline{\lim}\widetilde{T_n}$ comes from Lemma 3.6 (or Lemma 3.7). By Lemma 3.6 (or Lemma 3.7), we know that

$$\mu_\theta^{\mathbb{N}}(D_\theta) = 1.$$

On the other hand, by Lemma 2.11, we know that for each $\theta \in \Theta$, there exists a countable family of compact sets $(F_k^{(\theta)})_{k\in\mathbb{N}}$ such that

$$\mu_\theta^{\mathbb{N}}(\mathbb{R}^{\mathbb{N}} \setminus \cup_{k\in\mathbb{N}} F_k^{(\theta)}) = 0. \quad (4.5)$$

Finally, for $\theta \in \Theta$, we put

$$C_\theta = D_\theta \cap \cup_{k\in\mathbb{N}} F_k^{(\theta)}. \quad (4.6)$$

It is obvious that $(C_\theta)_{\theta\in\Theta}$ is a family of pairwise disjoint F_σ-sets such that

$$\mu_\theta^{\mathbb{N}}(C_\theta) = 1. \quad (4.7)$$

We put $T^\Diamond((x_k)_{k\in\mathbb{N}}) = \theta$ if $(x_k)_{k\in\mathbb{N}} \in C_\theta$ and $T^\Diamond((x_k)_{k\in\mathbb{N}}) = +\infty$ otherwise, where $(C_\theta)_{\theta\in\Theta}$ comes from Example 4.6.

Test 4.7. (The decision rule for a null hypothesis $H_0 = \theta = \theta_0(\theta_0 \in \mathbb{R})$

Null hypothesis: $H_0 : \theta = \theta_0$;

Alternative hypothesis: $H_1 : \theta \neq \theta_0$;

Test statistic: $T = T^\Diamond$;

Acceptance region for $H_0 : U_0 = \{\theta_0\}$;

Alternative critical region: $U_1 = \mathbb{R} \setminus \{\theta_0\}$.

Remark 4.8. The sum of errors of I and II types for Tests 4.7 is equal to zero (equivalently, Tests 4.7 is hypothesis testing of maximal reliability). Indeed,

$$\mu_{\theta}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in U_1 | H_0\}) = \mu_{\theta_0}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in \mathbb{R} \setminus \{\theta_0\}\} = \mu_{\theta_0}^{\mathbb{N}}(\mathbb{R}^{\mathbb{N}} \setminus D_{\theta_0}) = 0,$$

and

$$\mu_{\theta}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in U_0 | H_1\}) = \mu_{\theta}^{\mathbb{N}}(\{x : T^{\Diamond}(x) = \theta_0\}) = \mu_{\theta}^{\mathbb{N}}(D_{\theta_0}) = 0(\theta \neq \theta_0).$$

Test 4.9. (The decision rule for a countable competing hypotheses $\{H_i : \theta = \theta_i | i \in \mathbb{N} \ \& \ \theta_i \in \mathbb{R}\}$)

i-th Hypothesis: $H_i : \theta = \theta_i$;

Test statistic: $T = T^{\Diamond}$;

Acceptance region for $H_i : U_i = \{\theta_i\}$;

Alternative critical region: $V = \mathbb{R} \setminus \bigcup_{i \in \mathbb{N}} \{\theta_i\}$.

Remark 4.10. The sum of errors of I and II types for Tests 4.9 is equal to zero (equivalently, Tests 4.9 is a hypothesis testing of maximal reliability). Indeed,

$$\mu_{\theta}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in V | H_i\}) = \mu_{\theta_i}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in \mathbb{R} \setminus \bigcup_{i \in \mathbb{N}} \{\theta_i\}\}) = \mu_{\theta}^{\mathbb{N}}(\mathbb{R}^{\mathbb{N}} \setminus \bigcup_{i \in \mathbb{N}} D_{\theta_i}) = 0,$$

$$\mu_{\theta}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in U_i | H_j\}) = \mu_{\theta_j}^{\mathbb{N}}(\{x : T^{\Diamond}(x) = \theta_i\} = \mu_{\theta_j}^{\mathbb{N}}(D_{\theta_i}) = 0(i \neq j),$$

and

$$\mu_{\theta}^{\mathbb{N}}(\{x : T^{\Diamond}(x) \in \bigcup_{i \in \mathbb{N}} U_i | V\}) = \mu_{\theta}^{\mathbb{N}}(\bigcup_{i \in \mathbb{N}} D_{\theta_0}) = 0 \, (\theta \in \mathbb{R} \setminus \bigcup_{i \in \mathbb{N}} \{\theta_i\}).$$

Theorem 4.11. *For "almost every" infinite sample, a null hypothesis is rejected by Test* 4.7.

Proof. We have to show that a set of all infinite samples for which a Null Hypothesis is rejected by Test 4.7 is prevalence. Since the set C_{θ_0} is covered by the union of the countable family of compact sets $(F_k^{(\theta_0)})_{k\in\mathbb{N}}$, by Lemmas 2.4 and 2.5, we deduce that the Borel set C_{θ_0} as a subset of the Borel shy set $\bigcup_{k\in\mathbb{N}} F_k^{(\theta_0)}$ (see Definition 2.1) is also shy. The latter relation implies that $\mathbb{R}^{\mathbb{N}} \setminus C_{\theta_0}$ is prevalent because it is a complement of the shy set C_{θ_0}. This ends the proof of the theorem. □

Theorem 4.12. *For "almost every" infinite sample, all hypotheses* H_i $(i \in \mathbb{N})$ *are simultaneously rejected by Test* 4.9.

Proof. We have to show that a set of all infinite samples for which all hypothesis $H_i\,(i \in \mathbb{N})$ simultaneously are rejected by Test 4.9 is prevalence. Since for $i \in \mathbb{N}$, the set C_{θ_i} is covered by the union of the countable family of compact sets $(F_k^{(\theta_i)})_{k\in\mathbb{N}}$, by virtue of Lemmas 2.4-2.5, we deduce that a set C_{θ_i} as a subset of the Borel shy set $\bigcup_{k\in\mathbb{N}} F_k^{(\theta_i)}$ (see Definition 2.2) is also shy. By Lemma 2.5, we know that $\bigcup_{i\in\mathbb{N}} C_{\theta_i}$ is a shy set, which implies that $\mathbb{R}^{\mathbb{N}} \setminus \bigcup_{i\in\mathbb{N}} C_{\theta_i}$ as a complement of the shy set $\bigcup_{i\in\mathbb{N}} C_{\theta_i}$ is prevalent. This ends the proof of the theorem. □

Remark 4.13. An infinite sample average $\overline{X}_\infty : S \to \mathbb{R}$ is defined by

$$\overline{X}_\infty((x_k)_{k\in\mathbb{N}}) = \lim_{n\to\infty} \frac{\sum_{k=1}^{n} x_k}{n}, \tag{4.8}$$

for $(x_k)_{k\in\mathbb{N}} \in S$, where

$$S = \{(x_k)_{k\in\mathbb{N}} : (x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \ \&\ \text{there exists a finite limit } \lim_{n\to\infty} \frac{\sum_{k=1}^{n} x_k}{n}\}. \tag{4.9}$$

Notice that under condition of Lemma 3.6, we have the representation of S as the union of the family of pairwise disjoint Borel subsets $(S_\theta)_{\theta\in\mathbb{R}}$, where

$$S_\theta = \{(x_k)_{k\in\mathbb{N}}) : (x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}} \ \&\ \lim_{n\to\infty} \frac{\sum_{k=1}^n x_k}{n} = \theta\},$$

for $\theta \in \mathbb{R}$, and

$$(\forall\theta)(\theta \in \mathbb{R} \to \mu_\theta^{\mathbb{N}}(S_\theta) = 1).$$

Lemma 4.14. *S is a Borel shy set.*

Proof. It is obvious that S is a vector subspace of $\mathbb{R}^{\mathbb{N}}$. Indeed, if $(x_k)_{k\in\mathbb{N}}$ and $(y_k)_{k\in\mathbb{N}}$ are elements of S, then for $\alpha, \beta \in \mathbb{R}$, we get

$$\lim_{n\to\infty} \frac{\sum_{k=1}^n \alpha x_k + \beta y_k}{n} = \lim_{n\to\infty} \frac{\sum_{k=1}^n \alpha x_k}{n} + \lim_{n\to\infty} \frac{\sum_{k=1}^n \beta y_k}{n}$$

$$= \alpha \lim_{n\to\infty} \frac{\sum_{k=1}^n x_k}{n} + \beta \lim_{n\to\infty} \frac{\sum_{k=1}^n y_k}{n}, \tag{4.10}$$

which means that S is a vector subspace of $\mathbb{R}^{\mathbb{N}}$.

We have to show that S is a Borel subset of $\mathbb{R}^{\mathbb{N}}$.

For $i \in \mathbb{N}$, we denote by Pr_i the i-th projection on $\mathbb{R}^{\mathbb{N}}$ defined by

$$Pr_i((x_k)_{k\in\mathbb{N}}) = x_i, \tag{4.11}$$

for $(x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}$.

We put $S_n = \dfrac{\sum_{i=1}^{n} Pr_i}{n}$ for $n \in \mathbb{N}$. Then, on the one hand, the set of all infinite samples $x \in \mathbb{R}^{\mathbb{N}}$ for which there exists a finite limit $\lim_{n\to\infty} S_n(x)$ coincides with S. On the other hand, taking into account that $S_n : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is a continuous function for $n \in \mathbb{N}$ and the equality

$$S = \bigcap_{p=1}^{\infty} \bigcup_{n=1}^{\infty} \bigcap_{q=n}^{\infty} \bigcap_{m=1}^{\infty} \{x : x \in \mathbb{R}^{\mathbb{N}} \ \&\ |S_{q+m}(x) - S_q(x)| \le 1/p\} \tag{4.12}$$

holds, we claim that S is Borel subset of $\mathbb{R}^{\mathbb{N}}$.

We put $v = (1, 2, 3, \cdots)$. Let us show that v spans a line L such that every translation of L meets S at most one point; in particular, L is a probe for the complement of S. Indeed, assume the contrary. Then, there will be an element $(z_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}$ and two different parameters $t_1, t_2 \in \mathbb{R}$ such that $(z_k)_{k\in\mathbb{N}} + t_1(1, 2, 3, \cdots) \in S$ and $(z_k)_{k\in\mathbb{N}} + t_2(1, 2, 3, \cdots) \in S$. Since S is a vector space we deduce that $(t_2 - t_1)(1, 2, 3, \cdots) \in S$. Using the same argument, we claim that $(1, 2, 3, \cdots) \in S$ because $t_2 - t_1 \ne 0$, but the latter relation is false because

$$\lim_{n\to\infty} \frac{\sum_{k=1}^{n} k}{n} = \lim_{n\to\infty} \frac{n+1}{2} = +\infty. \tag{4.13}$$

This ends the proof of the lemma. □

We set $T^*((x_k)_{k\in\mathbb{N}}) = \overline{X}_\infty((x_k)_{k\in\mathbb{N}})$ if $(x_k)_{k\in\mathbb{N}} \in S \setminus S_0$, and $T^*((x_k)_{k\in\mathbb{N}}) = 0$ otherwise.

Then under conditions of Lemma 3.6, T^* is an infinite sample well-founded estimates of a parameter θ for the family $(\mu_\theta^{\mathbb{N}})_{\theta\in\mathbb{R}}$.

We consider the following:

Test 4.15. (The decision rule for all competing hypotheses $\{H_i : \theta = i : i \in \mathbb{R}\}$)

i-th Hypothesis: $H_i : \theta = i$;

Test statistic: $T = T^*$;

Acceptance region for $H_i : U_i = \{i\}$ for $i \in \mathbb{R}$;

Alternative critical region: $V = \emptyset$.

Remark 4.16. The sum of errors of I and II types for Tests 4.15 is equal to zero (equivalently, Tests 4.15 is a hypothesis testing of maximal reliability). Indeed,

$$\mu_\theta^{\mathbb{N}}(\{x : T^*(x) \in U_j | H_i\}) = \mu_i^{\mathbb{N}}(\{(T^*)^{-1}(j)\}) = \mu_i^{\mathbb{N}}(S_j) = 0,$$

for $i \neq j\,(i,\ j \in \mathbb{R})$.

Theorem 4.17. *For "almost every" infinite sample, a null hypothesis* $H_0 : \theta = 0$ *is accepted by Test* 4.15.

Proof. On the one hand, following Lemma 4.14, S is shy. On the other hand, the set of all infinite samples for which the H_0 hypothesis is accepted coincides with a set $(\mathbb{R}^{\mathbb{N}} \setminus S) \cup S_0$, which is also a prevalence (notice that S_0 as well S is shy). This ends the proof of the theorem. □

The next example of a hypothesis testing employs the result of Lemma 2.12 and the axiom of global choice [7].

Let τ be an operator of global choice. Let denote by $b(\mathbb{R})$ a set of all one-to-one mappings of $\mathbb{R}$ onto itself. We set $D_\theta = (C_{\tau(b(\mathbb{R}))(\theta)} \setminus S) \cup S_\theta$ for $\theta \in \mathbb{R}$, where $(C_t)_{t \in \mathbb{R}}$ comes from Lemma 2.12. Then $(D_\theta)_{\theta \in \mathbb{R}}$ will be Borel partition of $\mathbb{R}^{\mathbb{N}}$ such that D_θ is neither prevalent nor shy for $\theta \in \mathbb{R}$ and every translate of D_{θ_1} intersects D_{θ_2} in a shy set for different $\theta_1,\ \theta_2 \in \mathbb{R}$.

We put $T^{\star}((x_k)_{k\in\mathbb{N}}) = \theta$ if $(x_k)_{k\in\mathbb{N}} \in D_{\theta}$.

Under conditions of Lemma 3.6, $T^{\star}$ is an infinite sample well-founded estimate of a parameter θ for the family $(\mu_{\theta}^{\mathbb{N}})_{\theta\in\mathbb{R}}$.

Test 4.18. (The decision rule for all competing hypotheses $\{H_i : \theta = i : \ i \in \mathbb{R}\}$)

i-th Hypothesis: $H_i : \theta = i$;

Test statistic: $T = T^{\star}$;

Acceptance region for $H_i : U_i = \{i\}$ for $i \in \mathbb{R}$;

Alternative critical region: $V = \emptyset$.

Remark 4.19. The sum of errors of I and II types for Tests 4.18 is equal to zero (equivalently, Tests 4.18 is a hypothesis testing of maximal reliability).

Theorem 4.20. *The following assertions are valid*:

(i) *Unlike Test* 4.7, *there is no a null hypothesis which is rejected for* "*almost every*" *infinite sample by the Test 4.*18;

(ii) *Unlike Test* 4.9, *there is no a countable family of competing hypotheses* $\{H_k : \theta = i_k, i_k \in \mathbb{R}, k \in \mathbb{N}\}$ *such that for* "*almost every*" *infinite sample all null hypotheses are simultaneously rejected by Test* 4.18;

(iii) *Unlike Test* 4.15, *there is no a null hypothesis which is accepted for* "*almost every*" *infinite sample by the Test* 4.18.

Proof: Proof of the item (i). Let consider an arbitrary null hypothesis $H_0 =: \theta = i (i \in \mathbb{R})$. The set of infinite samples for which H_0 is rejected coincides with the set $\mathbb{R}^{\mathbb{N}} \setminus D_i$. By Lemma 2.12, we know that

C_i is not shy. Hence the set $D_i = (C_i \setminus S) \cup S_i$ also is non-shy because S and S_i are Borel shy sets (see Lemma 4.14). The latter relation means that complement of D_i is not prevalence and the validity of the item (i) is proved.

Proof of the item (ii). A set of all infinite samples for which all competing null hypotheses $\{H_k : \theta = i_k | k \in \mathbb{N}\}$ are simultaneously rejected coincides with the set $\mathbb{R}^{\mathbb{N}} \setminus \bigcup_{k \in \mathbb{N}} D_{i_k}$. Using the same argument used in the proof of the item (i), we claim that $\mathbb{R}^{\mathbb{N}} \setminus \bigcup_{k \in \mathbb{N}} D_{i_k}$ is not prevalence.

Proof of the item (iii). Let consider an arbitrary null hypothesis $H_0 : \theta = i (i \in \mathbb{R})$. The set of infinite samples for which H_0 is accepted coincides with the set D_i. By Lemma 2.12, we know that C_i is not prevalence. Hence the set $D_i = (C_i \setminus S) \cup S_i$ also is not prevalence because S and S_i are Borel shy sets (see Lemma 4.14). $\square$

We put

$$T((x_k)_{k \in \mathbb{N}}) = \overline{\lim} \widetilde{T_n}((x_k)_{k \in \mathbb{N}}) := \inf_n \sup_{m \geq n} \widetilde{T_m}((x_k)_{k \in \mathbb{N}}),$$

for $(x_k)_{k \in \mathbb{N}} \in \mathbb{R}^{\mathbb{N}}$, where $T_n((x_k)_{k \in \mathbb{N}}) = \dfrac{\sum_{k=1}^{n} x_k}{n}$.

Under conditions of Lemma 3.6, we consider the following tests:

Test 4.21. (The decision rule for a null hypothesis $H_0 : \theta = \theta_0 (\theta_0 \in \mathbb{R})$)

Null hypothesis: $H_0 : \theta = \theta_0$;

Alternative hypothesis: $H_1 : \theta \neq \theta_0$;

Test statistic: $T = \overline{\lim} \widetilde{T_n}$;

Acceptance region for $H_0 : U_0 = \{\theta_0\}$;

Alternative critical region: $U_1 = \mathbb{R} \setminus \{\theta_0\}$.

Test 4.22. (The decision rule for a countable competing hypotheses $\{H_i : \theta = \theta_i | i \in \mathbb{N} \ \& \ \theta_i \in \mathbb{R}\}$)

i-th Hypothesis: $H_i : \theta = \theta_i$;

Test statistic: $T = \overline{\lim}\widetilde{T_n}$;

Acceptance region for $H_i : U_i = \{\theta_i\}$;

Alternative critical region: $V = \mathbb{R} \setminus \bigcup_{i \in \mathbb{N}}\{\theta_i\}$.

Test 4.23. (The decision rule for all competing hypotheses $\{H_i : \theta = i : i \in \mathbb{R}\}$)

i-th Hypothesis: $H_i : \theta = i$;

Test statistic: $T = \overline{\lim}\widetilde{T_n}$;

Acceptance region for $H_i : U_i = \{i\}$ for $i \in \mathbb{R}$.

Alternative critical region: $V = \{\pm\infty\}$.

Remark 4.24. Notice that Tests 4.21-4.23 are statistical tests of maximal reliability. If we consider a set D of all infinite samples $(x_k)_{k \in \mathbb{N}} \in \mathbb{R}^{\mathbb{N}}$ for which $-\infty < \overline{\lim}\widetilde{T_n}((x_k)_{k \in \mathbb{N}}) < +\infty$, we observe that D is a vector subspace of $\mathbb{R}^{\mathbb{N}}$. By using the scheme of the proof of Lemma 4.14, we can easily prove that D is a Borel shy set in $\mathbb{R}^{\mathbb{N}}$.

Using Remark 4.24, one can get the validity of the following assertions:

Theorem 4.25. *For "almost every" infinite sample, a null hypothesis* $H_0 : \theta = \theta_0 (\theta_0 \in \mathbb{R})$ *is rejected by Test* 4.21.

Theorem 4.26. *For "almost every" infinite sample, all hypotheses* H_i $(i \in \mathbb{N})$ *are simultaneously rejected by Test* 4.22.

Theorem 4.27. *For "almost every" infinite sample, all hypotheses* H_i $(i \in \mathbb{R})$ *are simultaneously rejected by Test* 4.23.

5. Conclusion

Suppose that $T : \mathbb{R}^{\mathbb{N}} \to \overline{R}$ is an infinite sample well-founded estimate of a useful signal in the linear one-dimensional stochastic model (3.1). Notice that each hypothesis testing with such a test statistic is an infinite sample hypothesis testing of maximal reliability.

Main results of Section 4 motivate the following definition:

Definition 5.1. The estimate T is called subjective, if there is a null hypothesis which is accepted or rejected for "almost every" infinite sample by T under the assumption (3.3). Otherwise, the estimate T is called objective.

Definition 5.2. An infinite sample well-founded estimate $T : \mathbb{R}^{\mathbb{N}} \to \overline{R}$ is called strong objective if under the assumption (3.3) hold the following two conditions:

(i) $(\forall\theta)(\theta \in \mathbb{R} \to T^{-1}(\theta)$ is Haar ambivalent);

(ii) $(\forall\theta_1, \theta_2)(\theta_1, \theta_2 \in \mathbb{R} \to$ there exists an isometric (with respect to Tychonov metric) transformation $A_{(\theta_1, \theta_2)}$ of $\mathbb{R}^{\infty}$ such that $A_{(\theta_1, \theta_2)}(T^{-1}(\theta_1))$ $\Delta T^{-1}(\theta_2)$ is shy).

Notice that for objective infinite sample well-founded estimates the condition (i) of the Definition 5.2 authomatically holds.

Following Definition 5.1, the class of all infinite sample well-founded estimates of a useful signal in the linear one-dimensional stochastic model (3.1) can be divided into two pairwise disjoint non-empty subclasses: subjective and objective infinite sample well-founded estimates. For example, $T^{\diamond}$ (considered in Tests 4.7 and 4.9), T^{*} (considered in Test 4.15), and $\overline{\lim}\widetilde{T_n}$ (considered in Tests 4.21-4.23) are subjective infinite sample well-founded estimates. It can be shown that the test statistic $T^{\star}$ (considered in Test 4.18) is an objective infinite sample well-founded estimate.

Below we give an example of a strong objective infinite sample well-founded estimate in our model.

Example 5.3. Let J be an arbitrary subset of $\mathbb{N}$. We set

$$A_J = \{(x_i)_{i\in\mathbb{N}} : x_i \geq 0 \text{ for } i \in J \;\&\; x_i < 0 \text{ for } i \in \mathbb{N}\setminus J\}.$$

Then A_J is a Borel subset in $\mathbb{R}^{\mathbb{N}}$, which is a Haar ambivalent.

Let denote by $\mathcal{P}(\mathbb{N})$ power set of the set of all natural numbers and $H : \mathbb{R} \to \mathcal{P}(\mathbb{N})$ a bijection. We set $D_\theta = (A_{H(\theta)} \setminus S) \cup S_\theta$ for $\theta \in \mathbb{R}$, where S and $(S_\theta)_{\theta\in\mathbb{R}}$ comes from Remark 4. 13. Then $(D_\theta)_{\theta\in\mathbb{R}}$ will be Borel partition of $\mathbb{R}^{\mathbb{N}}$ such that D_θ is a Haar ambivalent for $\theta \in \mathbb{R}$ and for all $\theta_1, \theta_2 \in \mathbb{R}$, there exists an isometric (w.r.t. Tychonov metric) transformation $A_{(\theta_1,\theta_2)}$ of the $\mathbb{R}^{\infty}$ such that $A_{(\theta_1,\theta_2)}(D_{\theta_1})\Delta D_{\theta_2}$ is shy. We can define $A_{(\theta_1,\theta_2)}$ as follows: For $(x_k)_{k\in\mathbb{N}} \in \mathbb{R}^{\infty}$, we put $A_{(\theta_1,\theta_2)}((x_k)_{k\in\mathbb{N}}) = (y_k)_{k\in\mathbb{N}}$, where $y_k = -x_k$ if $k \in H(\theta_1)\Delta H(\theta_2)$ and $y_k = x_k$ otherwise. We put $T^{\circ}((x_k)_{k\in\mathbb{N}}) = \theta$ if $(x_k)_{k\in\mathbb{N}} \in D_\theta$.

Now it is not hard to show that under conditions of Lemma 3.6, T° is a strong objective infinite sample well-founded estimate of a parameter θ for the family $(\mu_{\theta}^{\mathbb{N}})_{\theta \in \mathbb{R}}$.

If we use in the statistical decision theory subjective well-founded estimates, then it will be natural to wait that practically all null hypotheses will be rejected for "almost every" infinite sample. The latter relation is consistent with Jum Nunnally conjecture [17] asserted that "*... in psychology, practically all null hypotheses are claimed to be false for sufficiently large samples* $\cdots$", and Jacob Cohen assertion [5] that "*it is usually nonsensical to perform an experiment with the sole aim of rejecting the null hypothesis*". The meaning of the fact that *Null Hypothesis was rejected for "almost every" infinite sample by some Hypothesis Testing of maximal reliability* can be explained by the phenomena that as usual only subjective infinite sample well-found estimates were under consideration in the theory of statistical decisions.

Notice that an application of the objective well-founded estimate in the same model shows us that above mentioned conjectures of Jum Nunnally [17] and Jacob Cohen [5] fail. It seems that the reduction of a big divergence between the theory of statistical decisions and results of statistical tests for sufficiently large samples directly depends on a choice of a reasonable element in the class of all objective infinite sample well-founded estimates.

Acknowledgement

The authors express thanks to anonymous referee for his careful reading of the paper manuscript and some helpful remarks.

The main results of Section 4 were reported on 16 January 2014 at the 42th Winter School in Abstract Analysis in Czech Republic (Svratka, Hotel Mane).

References

[1] R. Balka, Z. Buczolich and M. Elekes, Topological Hausdorff dimension and level sets of generic continuous functions on fractals, Chaos, Solitons and Fractals 45(12) (2012), 1579-1589.

[2] J. R. Christensen, Measure theoretic zero sets in infinite dimensional spaces and applications to differentiability of Lipschitz mappings, Actes du Deuxieme Colloque d'Analyse Fonctionnelle de Bordeaux (Univ. Bordeaux, 1973), I, pp. 29-39. Publ. Dep. Math. (Lyon) 10(2) (1973), 29-39.

[3] J. R. Christensen, Topology and Borel Structure, North-Holland Publishing Company, Amsterdam, 1974.

[4] S. L. Chow, Statistical Significance: Rationale, Validity and Utility, 1997.

[5] J. Cohen, The earth is round $(p < .05)$, American Psychologist 49(12) (1994), 997-1003.

[6] R. Dougherty, Examples of non-shy sets, Fund. Math. 144 (1994), 73-88.

[7] A. A. Fraenkel, Y. Bar-Hillel and A. Levy, Foundations of Set Theory, Second revised edition, With the collaboration of Dirk van Dalen, Studies in Logic and the Foundations of Mathematics Vol. 67, North-Holland Publishing Co., Amsterdam-London, 1973.

[8] H. L. Lavoie, S. A. Mulaik and J. H. Steiger, What if there were no significance tests?, Lawrence Erlbaum Associates, 1997.

[9] B. R. Hunt, T. Sauer and J. A. Yorke, Prevalence: A translation-invariant "almost every" on infinite-dimensional spaces, Bulletin (New Series) of the American Mathematical Society 27(2) (1992), 217-238.

[10] I. Sh. Ibramkhallilov and A. V. Skorokhod, On Well-off Estimates of Parameters of Stochastic Processes (in Russian), Kiev, 1980.

[11] A. B. Kharazishvili, Topological Aspects of Measure Theory (in Russian), Naukova Dumka, Kiev, 1984.

[12] R. Kline, Beyond Significance Testing: Reforming Data Analysis Methods in Behavioral Research, American Psychological Association, Washington, DC, 2004.

[13] L. Kuipers and H. Niederreiter, Uniform Distribution of Sequences, Wile-Interscience [John Wiley & Sons], New York-London-Sydney, 1974.

[14] D. N. McCloskey and T. Z. Stephen, The Cult of Statistical Significance: How the Standard Error Costs us Jobs, Justice, and Lives, University of Michigan Press, 2008.

[15] D. Morrison and R. Henkel, The Significance Test Controversy, Aldine Transaction, 2006.

[16] J. Mycielski, Some unsolved problems on the prevalence of ergodicity, instability, and algebraic independence, Ulam Quart. 1(3) (1992), 30 ff., approx. 8 pp.

[17] J. Nunnally, The place of statistics in psychology, Educational and Psychological Measurement 20(4) (1960), 641-650.

[18] M. Oakes, Statistical Inference: A Commentary for the Social and Behavioural Sciences, Wiley, Chichester New York, 1986.

[19] G. R. Pantsulaia, On a certain partition of the non-locally compact abelian Polish group $\mathbb{R}^{\mathbb{N}}$, Proc. A. Razmadze Math. Inst. 149 (2009), 75-86.

[20] Z. Zerakidze, G. Pantsulaia and G. Saatashvili, On the separation problem for a family of Borel and Baire G-powers of shift-measures on $\mathbb{R}$, Ukrainian Mathematical Journal 65(4) (2013), 470-485.

■